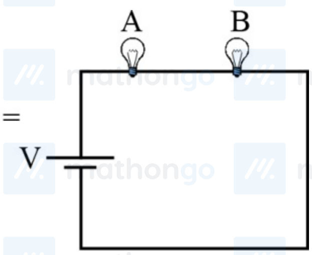
ANSWER KEYS

1. (3)	**2.** (4)	**3.** (3)	**4.** (4)	**5.** (3)	**6.** (3)
7. (2, 4)	**8.** (2)	**9.** (4)	**10.** (3)	**11.** (3)	**12.** (3)
13. (3)	**14.** (1)	**15.** (1)	**16.** (3)	**17.** (2)	**18.** (1)
19. (2)	**20.** (3)	**21.** (2)	**22.** (6)	**23.** (20)	**24.** (3)
25. (2)	**26.** (2)	**27.** (4)	**28.** (4)	**29.** (2)	**30.** (2)
31. (3)	**32.** (1)	**33.** (3)	**34.** (1)	**35.** (2)	**36.** (1)
37. (3)	**38.** (4)	**39.** (1)	**40.** (1)	**41.** (1)	**42.** (4)
43. (1)	**44.** (3)	**45.** (3)	**46.** (6)	**47.** (10)	**48.** (35)
49. (4051)	**50.** (18)	**51.** (3)	**52.** (2)	**53.** (4)	**54.** (1)
55. (2)	**56.** (3)	**57.** (4)	**58.** (2)	**59.** (3)	**60.** (4)
61. (1)	**62.** (4)	**63.** (3)	**64.** (2)	**65.** (2)	**66.** (4)
67. (3)	**68.** (1)	**69.** (3)	**70.** (2)	**71.** (53)	**72.** (81)
73. (9)	**74.** (3)	**75.** (2)			

1. **(3)** After the closing of S, C bulb or lamp become short-circuited

$\therefore$ new circuit =



the potential across A and B before the closing of S $= \frac{V}{3}$
Potential across A and B after the closing of S $= \frac{V}{2}$
As $P \propto V^2$

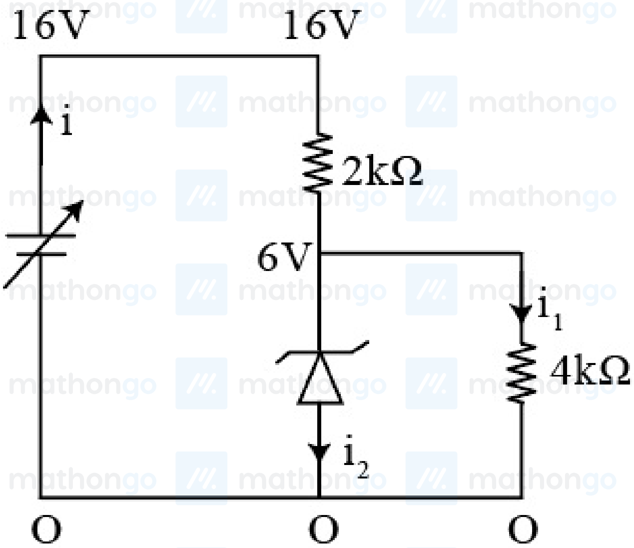
$\therefore \frac{P_i}{P_f} = \frac{4}{9}$

$P_f = \frac{P_i \times 9}{4} = 2.25\, P_i$

2. **(4)** $\frac{B^2}{2\mu_0} = \text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$

$\Rightarrow \frac{\left[ML^2T^{-2}\right]}{\left[L^3\right]} = \left[ML^{-1}T^{-2}\right]$

3. **(3)**



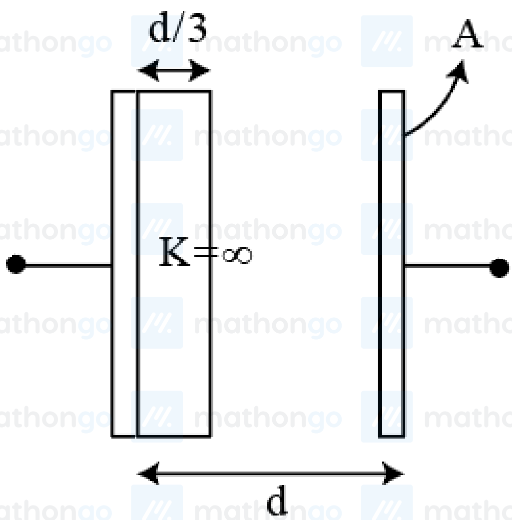
$i_1 = \frac{6}{4} = 1.5\,\text{mA}$

Maximum current will be obtained for battery voltage 16V

$i = \frac{16-6}{2} = 5\,\text{mA}$

$i_2(\max) = 5 - 1.5 = 3.5\,\text{mA}$

4.



(4)

$$C' = \frac{AG_0}{2d/3} = \frac{3}{2} \cdot \frac{AG_0}{d} = \frac{3C}{2}$$

$$U_i = \frac{1}{2} \cdot \frac{3C}{2}(100)^2$$

$$q_i = C'v = \frac{3C}{2} \times 100 = 150\text{C}$$

$$c_f = c$$

$$U_f = \frac{q_i^2}{2c} = \frac{(150c)^2}{2c} = \frac{150^2}{2} \cdot C$$

$$w = U_f - U_i = \left(\underset{1.125\times10^4}{\underset{\downarrow}{\frac{150^2}{2}}} - \underset{0.75\times10^4}{\underset{\downarrow}{\frac{3\times100^2}{4}}} \right) c = 93.75\mu\text{ J}$$

5. **(3)** We have

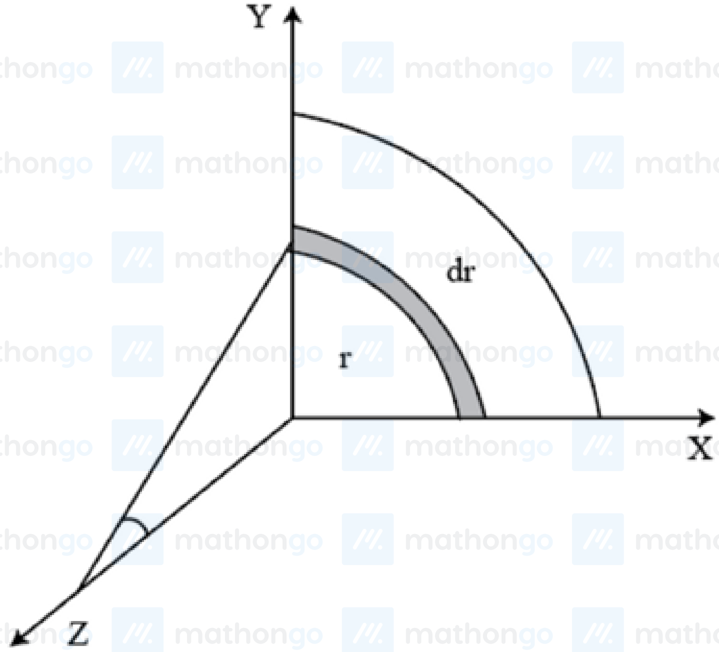
$p=\frac{F}{A}=\frac{F}{\pi R^2}$

So, $\frac{\Delta p}{p}\times 100=\frac{\Delta F}{F}\times 100+2\frac{\Delta R}{R}\times 100$

$=5+2(3)=5+6=11$

$\Rightarrow \frac{\Delta p}{p}\times 100=11\%$

6.



(3)

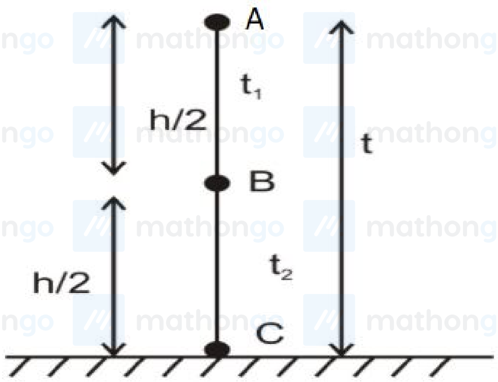
$V=\int_0^R \frac{k\alpha\pi r dr}{\alpha\sqrt{r^2+z^2}}=\frac{\sigma}{8\epsilon_0}\int_0^R \frac{rdr}{\sqrt{r^2+z^2}}$

$t^2=r^2+z^2$

$tdt=rdr$

$V=\frac{\sigma}{8\epsilon_0}\int_z^{\sqrt{2^2+R^2}} dt=\frac{\sigma}{8\epsilon_0}\left(\sqrt{R^2+z^2}-z\right)$

7.



(2, 4)

$-h = 0 - \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}} \ldots\ldots\ldots(1)$

$-\frac{h}{2} = 0 - \frac{1}{2}gt_1^2$

$t_1 = \sqrt{\frac{h}{g}} \ldots\ldots\ldots(2)$

so $t_2 = t - t_1$

$= \sqrt{\frac{2h}{g}} - \sqrt{\frac{h}{g}}$

$t_2 = \sqrt{\frac{h}{g}}\left(\sqrt{2}-1\right) \ldots\ldots\ldots(3)$

equation $3/1$

$\frac{t_2}{t_1} = \frac{\sqrt{\frac{h}{g}}\left(\sqrt{2}-1\right)}{\sqrt{\frac{h}{g}}}$

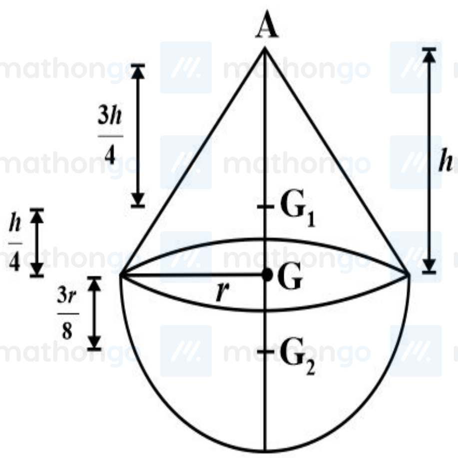
$\frac{t_2}{t_1} = \sqrt{2}-1$

8. **(2)** Volume of cone$= \frac{1}{3}\pi r^2 h$

Mass of cone, $m_1 = \rho \times \frac{1}{3}\pi r^2 h$

Mass of hemisphere, $m_2 = \rho \times \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \rho \times \frac{2}{3}\pi r^3$



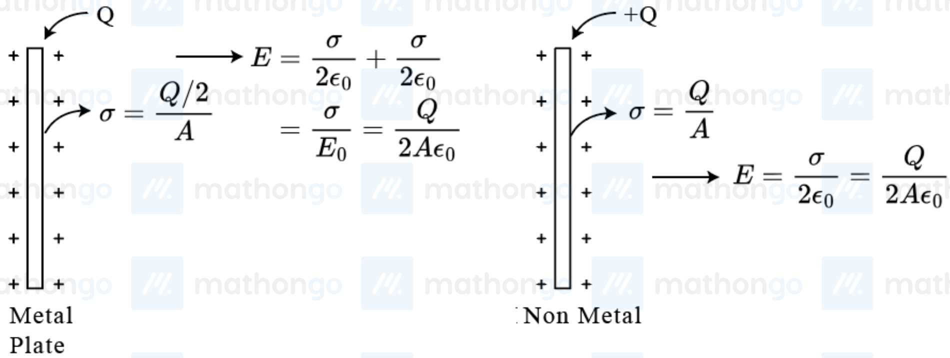
Now, $Y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$\Rightarrow 0=\frac{\rho\times\frac{1}{3}\pi r^2 h\times\frac{h}{4}+\frac{2}{3}\pi r^3\left(-\frac{3r}{8}\right)}{\rho\times\frac{1}{3}\pi r^2 h+\rho\times\frac{2}{3}\pi r^2}$

$\Rightarrow \rho\times\frac{1}{3}\pi r^3\left(\frac{h^2}{4}-2r\times\frac{3r}{8}\right)=0$

$\frac{h^2}{4}-\frac{3r^2}{4}=0$ or $h=\sqrt{3}\,r$

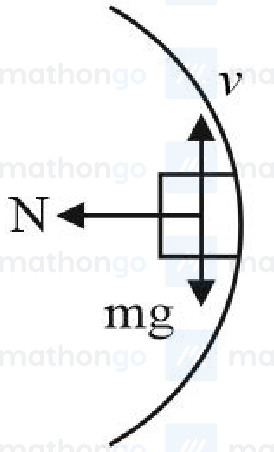
9.



(4)

10. (3)

The initial velocity of the block to undergo a complete vertical circular motion is $u=\sqrt{5gR}$



Let us assume that the speed of the block when it is moving in the vertical direction is v, then using conservation of mechanical energy we get

$\frac{1}{2}m\left(\sqrt{5gR}\right)^2=\frac{1}{2}mv^2+mgR$

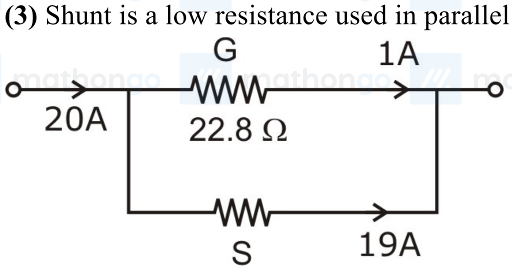
$\Rightarrow v^2=\sqrt{3gR}$

The tangential and centripetal accelerations of the block are

$a_t=g\ (\downarrow)$ and $a_c=3g\ (\leftarrow)$

$a_{net}=\sqrt{a_c^2+a_t^2}=g\sqrt{10}$

11. (3) Shunt is a low resistance used in parallel with the galvanometer to make it ammeter.



The voltage across galvanometer = voltage across the shunt

Given, $G = 22.8\ \Omega$, $i = 20\ A$, $i_G = 1\ A$

$\therefore S = \frac{1 \times 22.8}{20-1} = \frac{22.8}{19} = 1.2\ \Omega$

12. **(3)** $\because$ de-Broglie relation of a charged particles,

$\lambda = \frac{h}{mv}$

Velocity of charged particle at time t,

$v = v_0\hat{i} + \frac{eE_0}{m}t\hat{j}$ or $|v| = \sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}$

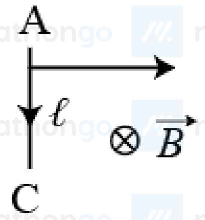
Hence, $\lambda = \frac{h}{m\sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}}$

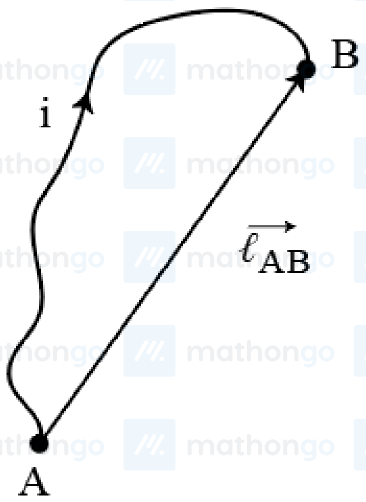
or

$$\lambda = \frac{h}{mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}} \text{ or } \lambda = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$$

$$\left(\because \lambda_0 = \frac{h}{mv_0}\right)$$

13.





(3)

$\overrightarrow{dF} = i \cdot (\overrightarrow{dl} \times \vec{B})$

$F = i\underbrace{\int \overrightarrow{dl}}_{\overrightarrow{l_{AB}}} \times \vec{B}$

$F = iBQ$

* Right hand Palm rule

14. **(1)** $g_e = g_p - R\omega^2 \quad \Rightarrow \quad \frac{g}{2} = g - R\omega^2$

$R\omega^2 = \frac{g}{2} \quad \Rightarrow \quad R^2\omega^2 = \frac{gR}{2} \quad \Rightarrow \quad V^2 = \frac{gR}{2}$ ...(1)

The escape velocity, $V_e = \sqrt{2gR}$ ...(2)

From (1) and (2)

$V_e = \sqrt{2 \times 2V^2} \quad \Rightarrow \quad V_e = 2V$

15. **(1)** $v_{avg} = \sqrt{\frac{8RT}{\pi M_1}}$

$v_{mp} = \sqrt{\frac{2RT}{M_2}}$

$\frac{8}{M_1\pi} = \frac{2}{M_2}$

$\frac{M_1}{M_2} = \frac{4}{\pi}$

16. **(3)**

The total binding energy,

$N^{14} = 7.5 \times 14$ MeV

$N^{15} = 7.7 \times 15$ MeV

The energy required to remove a neutron from N^{15},

$E = \text{BE}\ (N^{15}) - \text{BE}\ (N^{14})$

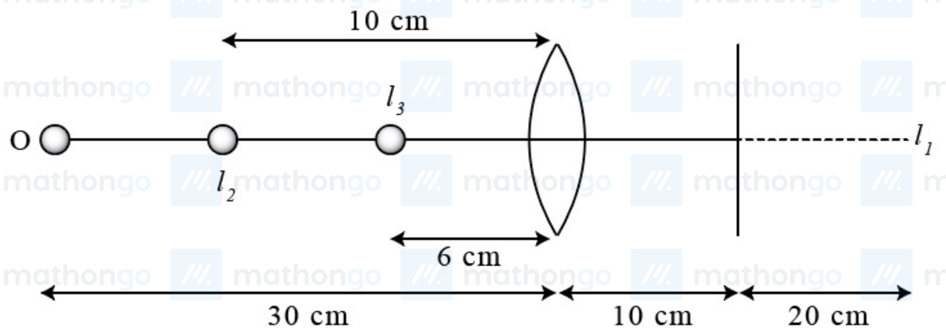
$E = (7.7 \times 15 - 7.5 \times 14)$ MeV

$\Rightarrow\ E = 10.5$ MeV

17. **(2)** Object is placed at distance $2f$ from the lens. So first image I will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.

Now applying lens formula

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$\therefore \frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$

or $v = 16$ cm

Therefore, the final image is at distance 16 cm from the mirror. But, this image will be real.

This is because ray of light is travelling from right to left.

18. **(1)** As $hR = \frac{2S}{\rho g} =$ a finite constant.

Hence when the tube is of insufficient length, radius of curvature of the liquid meniscus increases, so as to maintain the product hR a finite constant, i.e., as h decreases, R increases and the liquid meniscus becomes more and more flat, but the liquid does not overflow.

19. **(2)** From figure

$$\frac{V_4 - V_3}{V_2 - V_1} = \frac{P_3 - P_0}{P_0 - P_1} \Rightarrow \frac{V_4 - V_3}{10} = \frac{4 \times 10^5 - 3 \times 10^5}{3 \times 10^5 - 10^5}$$

$V_4 - V_3 = 5$ L

Now, work done

$W = \left(\frac{1}{2} \times 1 \times 2 \times 10^5 - \frac{1}{2} \times 5 \times 1 \times 10^5\right) \times 10^{-3} = 750\ J$

20. (3) f = a - bx

$f_{n_{ef}} = a - bx - f_s$

For block to move

at x = 0

$a - b_x > (f_s)_{max} = \mu mg$

$a > \mu mg$, So we are assuming motion starts at x = 0

$w = \int (a - bx - \mu mg)dx$

$o = ax - \frac{bx^2}{2} - \mu mgx$

$\Rightarrow x\left(a - \frac{bx}{2} - \mu mg\right) = 0$

x = 0

$a - \frac{bx}{2} - \mu mg = 0$

$\frac{bx}{2} = (a - \mu mg)$

$x = \frac{2}{b}(a - \mu mg)$

21. (2) Max ^{n} energy is liberated for transition

$E_n \rightarrow E_1$

and minimum energy for $E_n \rightarrow E_{n-1}$ Hence,

$\frac{E_1}{n^2} - \frac{E_1}{12} = 52.224eV$

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224eV$

Solving we get,

$E_1 = -54.4eV$

and $n = 5$

hence,

$E_1 = -\frac{13.6Z^2}{12} = -54.4$

$Z = 2.$

22. (6)

$e = -\frac{d\phi}{dt}$

$i = -\frac{1}{R}\frac{d\phi}{dt}$

$|i| = \frac{1}{3}\frac{d}{dt}\left(3at^3 - 3bt^2\right) = \frac{1}{3}\left[9at^2 - 6bt\right]$

$|i| = 3at^2 - 2bt = 6t^2 - 12t$

For maxima of i

$\frac{di}{dt} = 0$

$12t - 12 = 0 \Rightarrow t = 1sec$

$i_{max} = 6 \times 1^2 - 12 \times 1 = 6\ Amp.$

23. (20) Given: $d = 1.2\ mm,\ \lambda = 6000\ Å = 6 \times 10^{-7}\ m,\ D = 1\ m,\ x = 1\ cm = 1 \times 10^{-2}\ m$

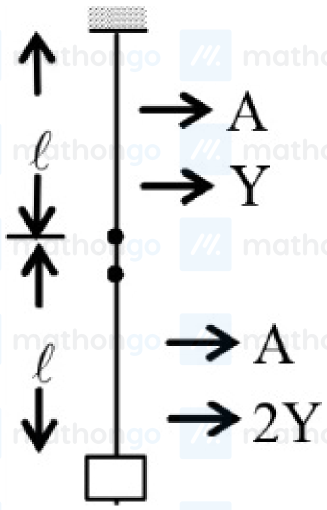
For n^{th} bright fringe, $x = n\frac{\lambda D}{d}$

$\therefore 1 \times 10^{-2} = \frac{n \times 6 \times 10^{-7} \times 1}{1.2 \times 10^{-3}}$

$\therefore n = \frac{1.2 \times 10^{-5}}{6 \times 10^{-7}} = 0.2 \times 10^{2} = 20$

There are 20 bright fringes formed over 1 cm width on the screen.

24. **(3)** The force constant of wires are



$\mathrm{k}_1 = \frac{YA}{l}$, $\mathrm{k}_2 = \frac{2YA}{l}$

In series $\mathrm{k} = \frac{\mathrm{k}_1 \mathrm{k}_2}{\mathrm{k}_1 + \mathrm{k}_2} = \frac{2}{3}\left(\frac{\mathrm{YA}}{l}\right)$

$\mathrm{T} = 2\pi\sqrt{\frac{3ml}{2YA}}$

25. **(2)** Drawing free body diagram of the cylinder with respect to plank.

$2\mu mg = ma \Rightarrow a = 2\mu g$

$(\mu mgR) = \frac{1}{2} mR^2 \alpha \,; \alpha - \frac{2\mu g}{r}$

Acceleration of point of contact with respect to plank is $4\mu g$

Velocity of pure rolling starts,

$-\mathrm{v} + 4\mu \mathrm{gt} = 0$

$\mathrm{t} = \frac{20}{4 \times 0.5 \times 10} = 1\mathrm{s}$

Distance traveled by cylinder with respect to plank in 1s is

$\mathrm{S}' = -\mathrm{vt} + \frac{1}{2}(2\mu \mathrm{g})\mathrm{t}^2 = -15\mathrm{m}$

At $\mathrm{t} = 1\mathrm{s}$, the velocity of cylinder with respect to plank is

$\mathrm{t} = 1\mathrm{s}$

$\mathrm{v}_{\mathrm{rel}} = -\mathrm{v} + 2\mu \mathrm{gt} = -20 + 2 \times 0.5 \times 10 = -10 \text{ m/s}$

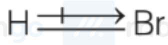
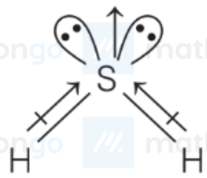
Remaining 10 m will be travelled in time $\mathrm{t}' = \frac{10}{10} = 1\mathrm{s}$

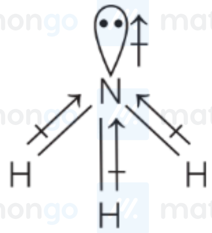
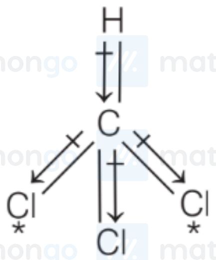
$\therefore$ Total time = 2s

26. **(2)** The correct match is : A-III, B-IV, C-V, D-I.

Dipole in different direction add up to give larger value.

Molecules with their geometry and dipole moments are given below

Molecule	Geometry	Dipole moment
	Linear	0.79
	Bent	0.95

Molecule	Geometry	Dipole moment
	Pyramidal	1.47
	Tetrahedral	1.04

($Cl \rightarrow$ 2 chlorine atoms are present opposite to each other, hence cancel dipole).

27. (4) In $CuCl_2$, Cu^{2+} has d^9 configuration, exhibit d-d transition and show colour. Similarly in $VOCl_2$, V^{4+} has d^1 configuration, can exhibit d-d transition and show colour.

$MnCl_2 =$ Pink colour

$Zn^{2+} \Rightarrow 3d^{10}$(Colourless)

$Hg^{2+} \Rightarrow 5d^{10}$(Colourless)

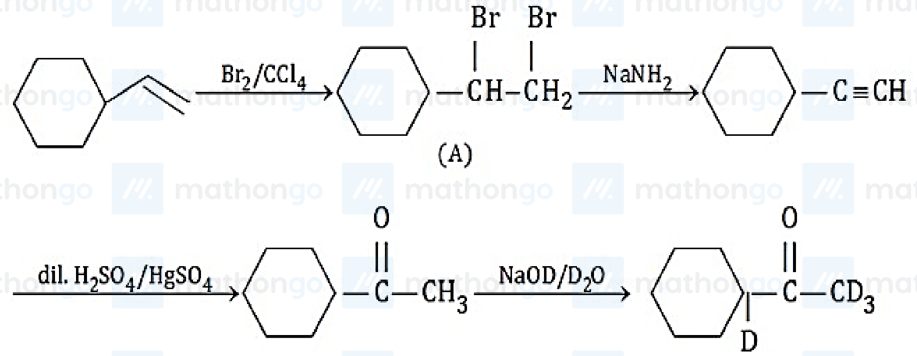
$V = 4s^2 3d^3$

$V^{4+} = 4s^0\ 3d^1$ (d - d transition)

$Cu^{+2} = d^9$ configuration = 1 - unpaired e^- electrons.

Since, in both $CuCl_2$ and $VOCl_2$, Cu^{2+} and V^{4+} ion contain 1 unpaired electron each, so their colour may be same.

28.



(4)

29. (2)

	N_2O_4	$\rightleftharpoons$	$2NO_2$
Initial moles	1		0
At equilibrium	$(1-0.2)$		$2\times 0.2 = 0.8$

Total moles $= 0.4 + 0.8 = 12$

$$K_P = \frac{\left(P_{NO_2}\right)^2}{\left(P_{N_2O_4}\right)} \Rightarrow K_P = \frac{\left(\frac{0.4}{1.2}\times 600\right)^2}{\left(\frac{0.8}{1.2}\times 600\right)} = 100$$

30. (2) I. $(-I)$ group, if are present along with $-COOH$ group in an acid, they increases the acidic property while (+) I groups or groups show hyperconjugation will decreases the acidic property.

(a) $NCCH_2COOH$: Contain, NC (i.e. $-I$) group along with $-COOH$, thus will increases the acidic nature.

(b) FCH_2-COOH : Contain, F (i.e. $-I$) group thus increases the acidic nature. Between $NCCH_2COOH$ and FCH_2COOH, $NCCH_2COOH$ is more acidic due to stronger (-) I effect.

(c) H_3CCH_2COOH : Contain, CH_3 group which show hyperconjugation effect and this is least acidic. Hence, the given order is correct.

II)(i) Structures which show more resonance structures or hyperconjugation among aldehydes are less reactive.

(ii) Aldehydes are more reactive than ketones.

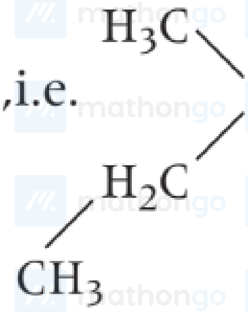
a) $CH_3\cdot CH_2\cdot CHO$ is an aldehyde in which CH_3-CH_2- group show hyperconjugation effect.

(b) $PhCOCH_3$, i.e. $\begin{matrix}Ph\diagdown \\ \quad CO \\ CH_3\diagup\end{matrix}$ is a ketone and is less reactive than $CH_3CH_2\cdot CHO$ and PhCHO (as these are aldehydes).

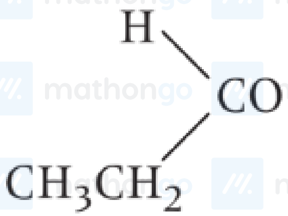
(c) PhCHO, i.e. $\begin{matrix}Ph\diagdown \\ \quad CO \\ H\diagup\end{matrix}$ is an aldehyde and is less reactive than CH_3CH_2-CHO due to resonance effect of Ph-group.

Thus, correct order is $CH_3CH_2\cdot CHO > PhCHO > PhCOCH_3$ and the given order is not correct.

(III) Boiling point of alcohols (due to ability to form H-bond) are higher than that of aldehydes which are more than that of ketones. (due to less steric-hinderance and more surface area in aldehyde group, having same number of C-atoms)

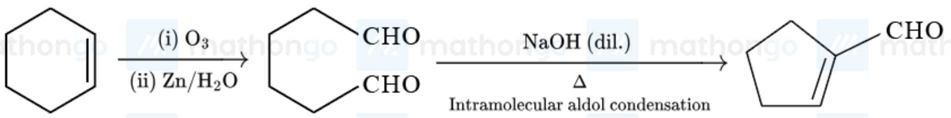
(a) $H_3C \cdot OC_2H_5$, i.e.  is an ether.

(b) $H_3C \cdot CH_2 \cdot CHO$ is an aldehyde, i.e.



(c) $H_3C - CH_2 - CH_2 - OH$ is an alcohol thus the given order for boiling point is correct. Hence, option (b) is the correct answer.

31. **(3)**



32. **(1)**

$$\begin{array}{lccc} & & S & 3S \\ Fe(OH)_3 \rightleftharpoons & & Fe^{3+} + & 3OH^- \end{array}$$

$K_{SP} = [Fe^{3+}][OH^-]^3$

$= (4.32 \times 10^{-2}](10^{-10})^3$

$= 4.32 \times 10^{-32}$

$PH = 4$

$[H^+] = 10^{-4}$

$[OH^-] = 10^{-10}$

Now in pure water, $[OH^-] = 10^{-7}$

$\therefore K_{SP} = [Fe^{3+}]\left[OH^{-7}\right]^3$

$4.32 \times 10^{-32} = S(10^{-7})^3$

$S = 4.32 \times 10^{-11}\ mol^{-1}$

Ratio $= \frac{10^{-2}}{10^{-11}} = 10^9$

33. **(3)**

Eq. wt of metal $= \frac{4}{17.75} \times 35.5 = 8$ g

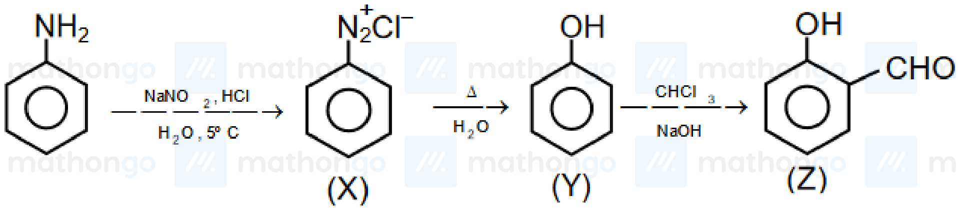
$M_2(CO_3)_x \longrightarrow M_2O_x$

Gram equivalents of carbonate = Gram equivalents of oxide

$$\frac{76}{(8+30)} = \frac{wt}{(8+8)}$$

$$wt = \frac{76 \times 16}{68} = 32$$

34. (1) Identify the Y and Z.



35. (2) Option A:

- Given Name: Pentaaaminethiocyanato-N-chromate(III) tetrachlorozincate(II)
- The compound is $\left[Cr(NH_3)_5\left(NCS'\right)\right][ZnCl_4]$:
- $\left[Cr(NH_3)_5(NCS)\right]$ is a cation with Chromium in the $+3$ oxidation state (hence "chromium(III)").
- NCS is named as thiocyanate with the donor atom specified as "N" (nitrogen-donor).

. $[ZnCl_4]$ is the anion named as "tetrachlorozincate(II)".

Option B:

- Mohr's salt is $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$:
- Mohr's salt is a double salt because it dissociates completely in water to give all ions Fe^{2+}, $(NH_4)^+$, SO_4^{2-}.
- Double salts retain their composition only in solid form and break into simple ions in solution.
- This statement is correct.

Option C:

- $\ln\left[Co(NH_3)_4(NO_2)_2\right]NO_3$:
- The complex ion is $\left[Co(NH_3)_4(NO_2)_2\right]^+$.
- The coordination number of cobalt is the number of ligand donor atoms attached to it.
- NH_3 contributes 4 donor atoms and NO_2 contributes 2 donor atoms, so the coordination number is $4+2=6$.
- This statement is correct.

Option D:

- $\ln[Fe(CO)_5]$:
- CO is a neutral ligand, and the oxidation state of Fe is 0 because the charge of the complex is neutral.
- Secondary valency refers to the coordination number (number of ligands attached).
- Here, Fe is coordinated with 5 ligands (CO), so its secondary valency is 5, not 0 .
- This statement is incorrect.

36. (1) Assertion:

"The electron gain enthalpy of N is + ve, while that of P is - ve."

This statement is true. Nitrogen's electron gain enthalpy is slightly positive $(+)$ because of its small size and high electron density, making it difficult for an additional electron to be accepted. In contrast, phosphorus has a larger size, reducing electron-electron repulsion, so it has a negative (-) electron gain enthalpy.

Reason:

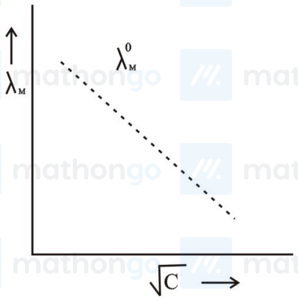
"This is due to the smaller atomic size of N, in which there is considerable electron-electron repulsion, and hence the additional electron is not accepted easily."

This statement is also true. The small size of nitrogen causes high electron density in its outermost shell, leading to significant repulsion when a new electron is added. This makes it less favorable to gain an electron, explaining why its electron gain enthalpy is positive.

37. (3)

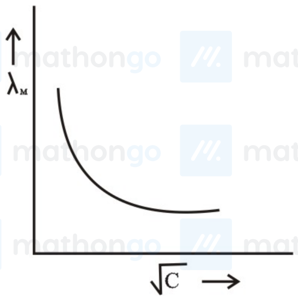
For strong electrolyte, curve given is straight line using Kohlrausch's Law of Independent Migration.

$\lambda_m = \lambda_m^0 - k\sqrt{C}$



As HCl is the strongest. (1) curve matches to HCl and (2) curve to NaCl.

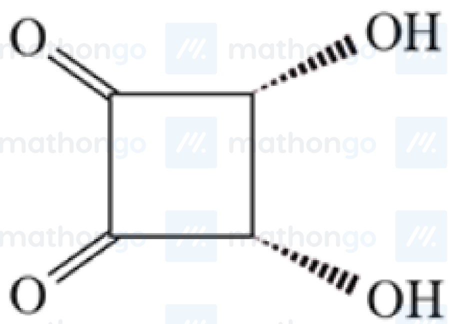
For weak electrolyte, when conc. decreases, α increases, the curve is given as



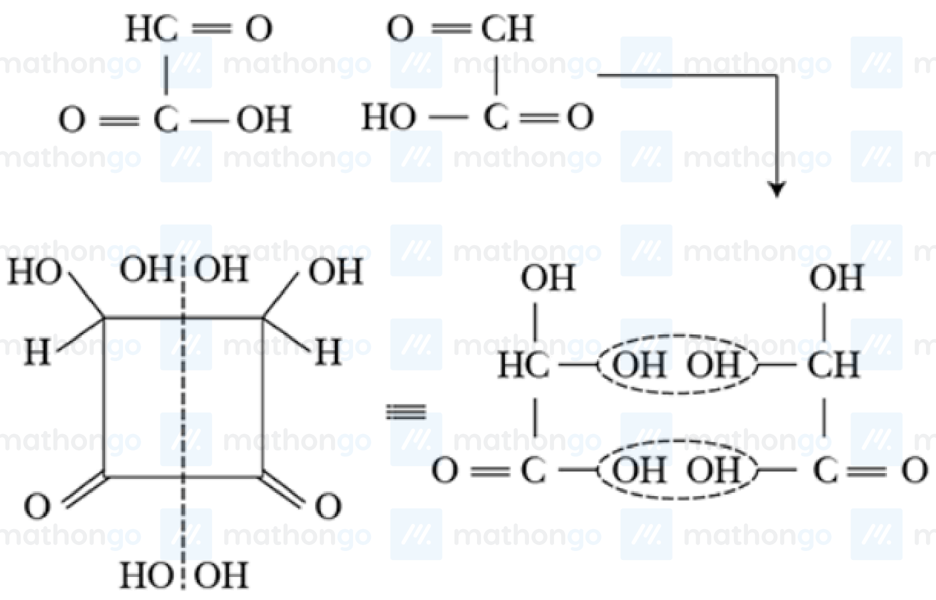
As NH_4 OH is weak base so (3) curve matches to NH_4 OH

$\therefore$ option (c) is the correct answer.

38. (4) Here the compound 'A' is



Glyoxalic acid is (OHC – COOH). Proceed reverse



39. (1)

Adsorption chromatography is a type of liquid chromatography in which chemicals are retained based on their adsorption and desorption at the support's surface, which acts as the stationary phase. This method is sometimes also known as liquid-solid chromatography. In this method, the mobile phase is either liquid or gaseous form. A stationary phase is a solid form. Paper chromatography is partition chromatography.

40. (1) $[NiCl_4]^{2-}$: Tetrahedral and paramagnetic (weak field ligand, 2 unpaired electrons).

$[Ni(CN)_4]^{2-}$: Square planar and diamagnetic (strong field ligand, all electrons paired).

$\left[Cu(NH_3)_4\right]^{2+} \Leftarrow dsp^2$, square planer and paramagnetic

$[Ni(CO)_4] \Leftarrow$ is sp^3, tetrahedral but diamagnetic

41. (1)

Due to inert pair effect Pb has four electrons in its valence shell but it shows $+2$ oxidation state. In other words due to inert pair effect $+2$ oxidation state is more stable than $+4$ of Pb.

So, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

42. (4) As compound having free anomeric carbon is reducing in nature. Therefore D is non-reducing as in it there is no free anomeric carbon.

To identify the non-reducing sugar, we need to consider whether the sugar has a free aldehyde or ketone group capable of acting as a reducing agent. If both anomeric carbons are involved in a glycosidic bond, the sugar is non-reducing.

1. Option A: This sugar has a free anomeric carbon (no glycosidic bond involving all reactive groups), so it is a reducing sugar.

2. Option B: Contains methyl groups blocking the reducing ends, so it appears non-reducing. However, its structure suggests a non-common disaccharide.

3. Option C: This is glucose, which has a free aldehyde group (in equilibrium with open-chain form), making it a reducing sugar.

4. Option D: Both anomeric carbons are involved in a glycosidic bond (e.g., in sucrose), so it has no free aldehyde or ketone group. This is a non-reducing sugar.

43. (1) Maximum number of electrons

$= 2n^2 = 2 \times 4 \times 4 = 32(\text{ when } n = 4)$

Half of the electrons have $\left(+\frac{1}{2}\right)$ spin, i.e., 16

44. (3)

$Fe(s) \rightarrow Fe^{2+} + 2e^-; \Delta G_1^o$

$2H^+ + 2e^- + \frac{1}{2}O_2 \rightarrow H_2O(l); \Delta G_2^o$

$Fe(s) + 2H^+ + \frac{1}{2}O_2 \rightarrow Fe^{2+} + H_2O; \Delta G_3^o$

Applying, $\Delta G_1^o + \Delta G_2^o = \Delta G_3^o$

$\Delta G_3^O = (-2\,F \times 0.44) + (-2\,F \times 1.23)$

$\Delta G_3^o = (-2 \times 96500 \times 0.44) + (-2 \times 96500 \times 1.23)$

$\Delta G_3^o = -322310\ J$

$\therefore \Delta G_3^o = -322\ kJ$

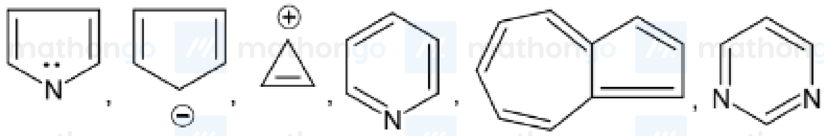
45. (3) (a) Higher the charge / size ratio, Higher is the polarising power. Given order is correct.

(b) Geometry for both the species is tetrahedral, as in both ' O ' is sp^3 hybridised

(c) $113 = [Rn]5f^{14}6d^{10}7s^27p^1 =$ Due to inert pair effect $7s^2$ electron do not take part in bond formation, so, most stable oxidation state is $+1$.

(d) Successive ionization energies are higher for every element. 2^{nd} ionization of K happens from $3s^23p^6$ configuration which is an inert gas configuration and for Ca it happens from $4s^1$. So, K has higher second ionisation energy than Ca.

46. (6) Aromatic compounds



47. (10)

$t = \frac{[R_0] - [R]_E}{K} \quad t = \frac{0.1 - 0.075}{2.5 \times 10^{-3}}$

$= \frac{0.025}{2.5 \times 10^{-3}} t = 0.01 \times 10^3 = 10\ s$

48. (35) W_B mass of ethylene glycol $(M = 62) = 46.5gm$

W_A mass of water $= 160gm$

ΔT_f depression in freezing point $= 11.16\ K$

Let W_B be the mass of water remaining

$\Delta T_f = K_t \frac{W_B \times 1000}{M_B\ W_B'}$

$11.16 = \frac{1.86 \times 46.5 \times 1000}{62 \times W_B'}$

$\therefore W_B' = 125gm$

Mass of ice that separates out on cooling

$= W_B - W_B = 160 - 125 = 35gm$

49. (4051) Writing the formation reaction

$4C(s) + 2H_2(g) + \frac{1}{2}O_2(g) \rightarrow C_4H_4O(g)$

$\Delta H_f^\circ = -62\cdot 0 = \Sigma_{\text{reacants}}$ Enthalpy of atomisation $- \Sigma_{\text{products}}$ Enthalpy of atomisation

Σ_{products} Enthalpy of atomisation $= \Sigma_{\text{reactants}}$ Enthalpy of atomisation $+ 62\cdot 0 = 4\times 717 + 4\times 218 + 249 + 62$

$= 2868 + 872 + 249 + 62 = 4051\ \text{kJ mol}^{-1}$

50. (18) The reaction is as follows

$$\underset{}{NaCl} \xrightarrow{K_2Cr_2O_7/H^{\oplus}} \underset{(A)}{CrO_2Cl_2} \xrightarrow{NaOH} \underset{(B)}{Na_2CrO_4} \xrightarrow[H^{\oplus}]{H_2O_2} \underset{(C)}{CrO_5}$$

$CrO_2Cl_2 \rightarrow 5$

$Na_2CrO_4 \rightarrow 7$

$CrO_5 \rightarrow 6$, Total $= 18$

51. (3)

$ar^5 = 4(ar^3) \Rightarrow r^2 = 4 \Rightarrow r = 2$

$a(2)^8 - a(2^6) = 192 \Rightarrow a = 1$

$S_n - S_3 = 1016$

$(2^n - 1) - (1 + 2 + 4) = 1016$

$2^{10} = 1024 \Rightarrow n = 10$

52. (2) (i) $-1 \le 2x - x^2 \le 1$ (for $\sin^{-1}$ to be defined)

$\Rightarrow -1 \le x^2 - 2x \le 1$

i.e. $x^2 - 2x + 1 \ge 0$ and $x^2 - 2x - 1 \le 0$

$(x-1)^2 \ge 0$ and $(x-1)^2 - \left(\sqrt{2}\right)^2 \le 0$

$x \in R$ and $\left(x - 1 - \sqrt{2}\right)\left(x - 1 + \sqrt{2}\right) \le 0$

$\Rightarrow x \in \left[1 - \sqrt{2}, 1 + \sqrt{2}\right]$ … (1)

(ii) $2 - \frac{1}{|x|} \ge 0 \Rightarrow \frac{1}{|x|} \le 2 \Rightarrow |x| \ge \frac{1}{2} \Rightarrow x \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ … (2)

(iii) $[x^2] \ne 0 \Rightarrow x^2 \notin [0,1)$

$\Rightarrow x \notin (-1,1) \Rightarrow x \in (-\infty, -1] \cup [1, \infty)$ … (3)

Hence, $(1)\cap(2)\cap(3)$

$\Rightarrow x \in \left[1, 1 + \sqrt{2}\right]$

53. (4) Let X denotes the number of tosses required.

$P(X \ge 4) = \left(\frac{2}{3}\right)^3 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^4 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^5 \cdot \frac{1}{3} + \ldots\ldots \infty = \frac{\left(\frac{2}{3}\right)^3 \cdot \frac{1}{3}}{1 - \frac{2}{3}} = \frac{8}{27}$

$P(X \ge 7) = \left(\frac{2}{3}\right)^6 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^7 \cdot \frac{1}{3} + \ldots\ldots \infty = \left(\frac{\left(\frac{2}{3}\right)^6 \cdot \frac{1}{3}}{1 - \frac{2}{3}}\right) = \left(\frac{2}{3}\right)^6$

Required probability $= \frac{\left(\frac{2}{3}\right)^6}{\left(\frac{2}{3}\right)^3} = \frac{8}{27}$

54. (1) $\lim_{x\to 0}\left\{1 + x\log\left(1 + a^2\right)\right\}^{1/x}$

$= 2a\sin^2\theta, a > 0$ and $\theta \in R$

$\lim_{x\to 0}\left\{1 + x\log\left(1 + a^2\right)\right\}^{1/x}$ is of the form 1^∞

$= e^{\lim_{x\to 0}\frac{1}{x}\left\{1 + x\log\left(1 + a^2\right) - 1\right\}} = e^{\lim_{x\to 0}\log\left(1 + a^2\right)} = 1 + a^2$

$\Rightarrow 1 + a^2 = 2a\sin^2\theta \Rightarrow a^2 - 2a\sin^2\theta + 1 = 0$

$$\Rightarrow \quad a=\frac{2\sin^2\theta \pm \sqrt{4\sin^4\theta-4}}{2}$$

$$a=\frac{2\sin^2\theta \pm \sqrt{4\left(\sin^4\theta-1\right)}}{2}$$

$$a=\sin^2\theta \pm \sqrt{\sin^4\theta-1}$$

$\sin^4\theta-1 \geqslant 0$

$\Rightarrow \sin^4\theta=1$

$\sin^2\theta=1=\sin^2\frac{\pi}{2}$

$\Rightarrow \quad \theta=n\pi \pm \frac{\pi}{2}$

55. (2) Given, $f(x+y)=f(x)\cdot f(y)$

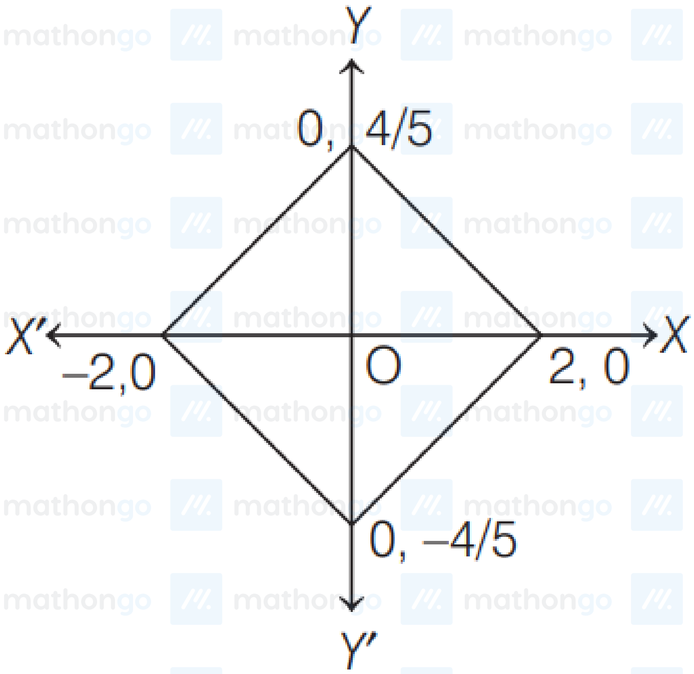
$\therefore \quad f(x)=a^x$

$f(1)=a^1=2 \Rightarrow a=2$

$\therefore \quad f(x)=2^x$

Area enclosed by the lines $2|x|+5|y| \leq 4$ is

$=4\left(\frac{1}{2}\times 2\times \frac{4}{5}\right)=\frac{16}{5}$



$=\frac{16}{1+(2)^2}=\frac{(2)^4}{1+(2)^2}=\frac{f(4)}{1+f(2)}$

56. (3) Let $z=x+iy$

$|z-4|=|z-8| \Rightarrow x=6$

$3|(x-12)+yi|=5|x+(y-8)i|$
$9\left[36+y^2\right]=25\left[36+(y-8)^2\right]$ (substituting $x=6$)
$9\cdot 36+9y^2=25\cdot 36+25\left[y^2+64-16y\right]$
$16y^2-25\cdot 16y+36\cdot 16+25\cdot 64=0$
$y^2-25y+36+100=0$
$y^2-25y+136=0$
$(y-17)(y-8)=0$
then, $y=17$ or $y=8$

57. **(4)** Since the ellipse contains the circle
$\therefore$ Solving circle with ellipse, we get
$b^2x^2+a^2\left(1-(x-1)^2\right)=a^2b^2$
$\left(b^2-a^2\right)x^2+2a^2x-a^2b^2=0$
$D=0$
$4a^4+4\left(a^2b^2\right)\left(b^2-a^2\right)=0$
$a^2+b^2\left(b^2-a^2\right)=0$
$a^2-b^2\left(a^2e^2\right)=0$
$1=b^2e^2 \Rightarrow be=1$
Now,
area of ellipse, $A=\pi ab$
$A^2=\pi^2a^2b^2$
$e^2=1-\frac{b^2}{a^2}=\frac{1}{b^2}$
$e^2=1-\frac{1}{b^2}=\frac{b^2}{a^2}$
$a^2=\frac{b^4}{b^2-1}$
Now,
$A^2=f(b)=\frac{b^6}{b^2-1}$
For maxima and minima $\quad f'(b)=0$
$\left(b^2-1\right)6b^5-b^6(2b)=0 \Rightarrow 3\left(b^2-1\right)=b^2$
$b^2=\frac{3}{2};\quad \therefore \quad a^2=\frac{9}{2}$
$\therefore \quad a^2+b^2=\frac{9}{2}+\frac{3}{2}=6=2n \quad \Rightarrow \quad n=3$

58. Let $\left(\frac{x}{2},\frac{x}{2},\frac{y}{3},\frac{y}{3},\frac{y}{3},\frac{z}{4},\frac{z}{4},\frac{z}{4},\frac{z}{4}\right)$
$\therefore \quad$ GM $\leq$ AM

(2) $\Rightarrow \quad \left(\frac{x^2\cdot y^3\cdot z^4}{3^3\cdot 2^{10}}\right)^{\frac{1}{9}}\leq 3$
$\Rightarrow \quad x^2y^3z^4\leq 3^{12}\cdot 2^{10}=9\cdot 6^{10}$

59. **(3)** $|\mathrm{A}|^{2^4}=(2\times 5)^{16}\Rightarrow |\mathrm{A}|=\pm 10$
$\because |\mathrm{A}|=x+y+z$, where $x,y,z\in \mathrm{N}$

$\therefore x+y+z=10$

$\therefore$ number of solutions $= {}^{10-1}\mathrm{C}_{3-1} = {}^{9}\mathrm{C}_2 = \frac{9\times 8}{2} = 36$

60. $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k},\quad \vec{b}=y\hat{i}-2z\hat{j}+3x\hat{k},\quad \vec{c}=2z\hat{i}+3x\hat{j}-y\hat{k}.$

(4) $\vec{a}\cdot\vec{d}=0,\quad \vec{d}=\hat{i}-\hat{j}+2\hat{k}.$

$x-y+2z=0.\quad \ldots(1)$

$|\vec{a}|=2\sqrt{3},\quad x^2+y^2+z^2=12.\quad \ldots(2)$

From (1), $x=y-2z$. Substituting into (2):

$(y-2z)^2+y^2+z^2=12$

$2y^2-4yz+5z^2=12\quad \ldots(3)$

Let $z=2$, substituting into (3):

$2y^2-8y+20=12$

$y^2-4y+4=0,\quad y=2.$

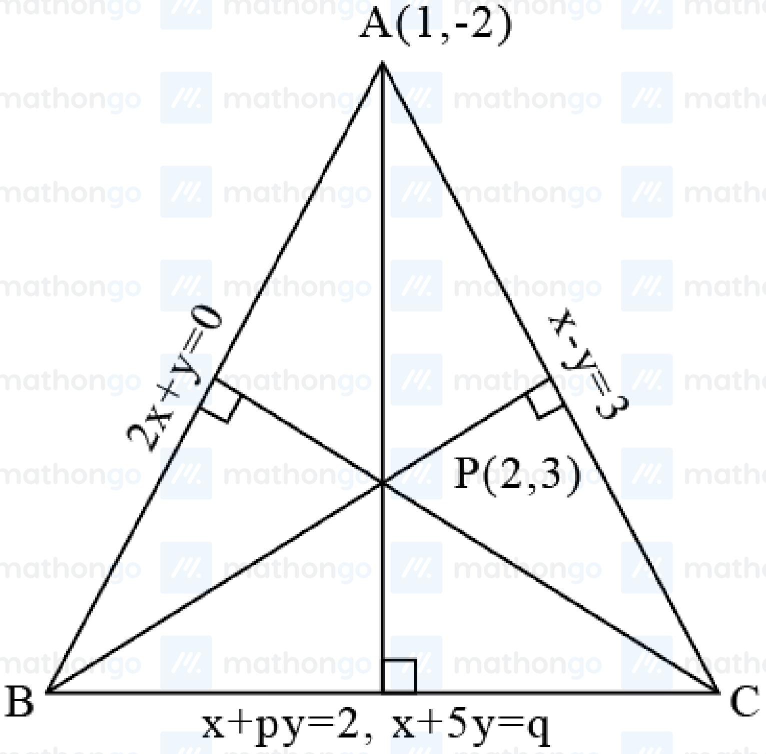
From (1), $x=-2$.

$\vec{a}\cdot\vec{b}=xy-2yz+3zx$

$\vec{a}\cdot\vec{b}=(-2)(2)-2(2)(2)+3(2)(-2)=-24.$

61. (1) In $\triangle ABC$

Equation of AB, BC and CA are $2x+y=0$, $x+py=q$ and $x-y=3$ respectively and $P(2,3)$ is orthocentre.



Solving $2x+y=0$ and $x-y=3$
$y=-2x$ and $x+2x=3$
$\Rightarrow \quad 3x=3$
$\Rightarrow \quad x=1, y=-2$
$\therefore A(1,-2)$
Slope of $AP=\dfrac{3-(-2)}{2-1}=5$
Slope of $BC=-\dfrac{1}{p}$
$AP \perp BC$
$\Rightarrow \quad 5\times\left(-\dfrac{1}{p}\right)=-1$
$p=5$
Now for vertex B
$2x+y=0, y=-2x$
$x+5y=q$
$x-10x=q \Rightarrow x=-\dfrac{q}{9}, y=\dfrac{2q}{9}$
B is $\left(-\dfrac{q}{9},\dfrac{2q}{9}\right)$
Slope of $BP=\dfrac{\frac{2q}{9}-3}{-\frac{q}{9}-2}=\dfrac{2q-27}{-q-18}$
Slope of $AC=1$
$BP\perp AC \Rightarrow \left(\dfrac{q-27}{-q-18}\right)(1)=-1$
$\Rightarrow \quad 2q-27=q+18$
$2q-q=18+27$
$q=45$
$p+q=5+45=50$

62. **(4)**
$$I=2\int_0^1\left(\tan^{-1}\left[x^2+\frac{1}{2}\right]+\cot^{-1}\left[x^2-\frac{1}{2}\right]\right)dx$$
$$=2\left[\int_0^{1/\sqrt{2}}\left(\tan^{-1}0+\cot^{-1}(-1)\right)dx\int_{1/\sqrt{2}}^{1}\left(\tan^{-1}1+\cot^{-1}0\right)dx\right]=\frac{3\pi}{2}$$

63. **(3)** For observations $x_1,\ x_2,\ \ldots\ldots\ldots\ldots x_{50}$
Mean, $\bar{x}=\frac{\sum x_i}{50}=16\ldots\ldots$(i)
Variance, $\sigma^2=\frac{\sum x_i^2}{50}-\left(\bar{x}\right)^2=16^2$
$\Rightarrow \frac{\sum x_i^2}{50}=16^2+\left(\bar{x}\right)^2=16^2+16^2=512\ldots\ldots$(ii)
So, the mean value of $(x_1-4)^2,\ (x_2-4)^2,\ \ldots\ldots(x_{50}-4)^2$ will be
$=\frac{\sum(x_i-4)^2}{50}=\frac{\sum x_i^2-8\sum x_i+16\times 50}{50}$
$=\frac{\sum x_i^2}{50}-8\frac{\sum x_i}{50}+16=512-8\times 16+16=400$ (using (i) and (ii))

64. **(2)** $\frac{1}{1+y^2}\cdot\frac{dy}{dx}+2x\left(\tan^{-1}y\right)=x^3$
Put $\tan^{-1}y=z$

$\therefore \frac{1}{1+y^2} \cdot \frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dz}{dx} + (2x)z = x^3 \quad \Rightarrow \quad z \cdot e^{x^2} = \frac{1}{2}\int 2e^{x^2} \cdot x^3 dx + c$

Put $x^2 = \mathrm{t}$

$\Rightarrow \quad \left(\tan^{-1} y\right) e^{x^2} = \frac{1}{2}\int e^t \cdot t dt + c$

$\Rightarrow e^{x^2}\left(\tan^{-1} y\right) = \frac{1}{2}\left\{e^t \cdot t - e^t\right\} + c$

$\Rightarrow \quad 2e^{x^2} \tan^{-1} y = x^2 e^{x^2} - e^{x^2} + 2c$

$\Rightarrow \quad 2\tan^{-1} y = x^2 - 1 + 2ce^{-x^2}$

65. (2)

Point of intersection of lines $x = y = z$ and $x = \frac{y}{2} = \frac{z}{3}$ is $(0,\ 0,\ 0)$

Angle between lines is $\cos\theta = \frac{(1)(1)+(1)(2)+(1)(3)}{\sqrt{1+1+1}.\sqrt{1+2^2+3^2}} = \frac{6}{\sqrt{42}}$

$\Rightarrow \sin\theta = \frac{\sqrt{6}}{\sqrt{42}}$

Let a point on line $x = \frac{y}{2} = \frac{z}{3}$ is $A(\lambda,\ 2\lambda,\ 3\lambda)$

$x = y = z$ passes through $B(1,1,1)$ and third line also passes through $B(1,\ 1,\ 1)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}OA.\,OB\sin\theta = \frac{1}{2}\sqrt{3}\ \lambda\sqrt{14}\ \frac{\sqrt{6}}{\sqrt{42}} = \sqrt{6}$

$\therefore\ \lambda = 2$

$\therefore$ Required coordinate $= (2,\ 4,\ 6)$.

66. (4)

For any relation R defined on set A is said to be reflexive if $(a,a) \in R\ \forall\ a \in A$, symmetric if $(a,b) \in R \Leftrightarrow (b,a) \in R\ \forall\ a,\ b \in A$ and transitive if $(a,b) \in R,\ (b,c) \in R$, then $(a,c) \in R$ for all $a,\ b,\ c \in A$.

Given,

$(\mathrm{x},\mathrm{y}) \Leftrightarrow \mathrm{x}^2 - 8\mathrm{xy} + 7\mathrm{y}^2 = 0$

So,

$(\mathrm{x},\mathrm{x}) \Leftrightarrow \mathrm{x}^2 - 8x \times x + 7\mathrm{x}^2 = 0$, this is true. So, the relation is reflexive.

Again, $(\mathrm{y},\mathrm{x}) \Leftrightarrow \mathrm{y}^2 - 8\mathrm{xy} + 7\mathrm{x}^2 \neq \mathrm{x}^2 - 8\mathrm{xy} + 7\mathrm{y}^2$

Hence, the relation is not symmetric.

Again,

$(\mathrm{x},\mathrm{y}) \Leftrightarrow \mathrm{x}^2 - 8\mathrm{xy} + 7\mathrm{y}^2 = 0 \ldots(1)\ \&\ (\mathrm{y},\mathrm{z}) \Leftrightarrow \mathrm{y}^2 - 8zy + 7\mathrm{z}^2 = 0 \ldots.(2)$

So, from the equation (1) & (2), we cannot determine

$xRy\ \&\ yRz \Rightarrow xRz \Leftrightarrow \mathrm{x}^2 - 8xz + 7\mathrm{z}^2 = 0$

So, the relation is not transitive.

67. **(3)**

$$f(x)=\begin{cases}\frac{1}{x}; & \text{if } x^2>1 \Rightarrow x<-1 \quad \text{or } x>1\\ ax^3+bx^2; & \text{if } 0\le x^2<1 \Rightarrow -1<x<1\\ \frac{1/x+ax^3+bx^2}{2}; & \text{if } x^2=1\end{cases}$$

$\because \quad f$ is continuous

$\therefore \quad$ at $x=1 \quad 1=a+b\ldots(1)$

and at $x=-1$

$\therefore \quad b=0 \quad \dfrac{-1=-a+b}{\cdot}\ldots(2)$ and $a=1$

$\therefore \quad$ point A and B are $=(-1,3)$ and $(1,-1)$.

$\therefore \quad g'(x)=\lambda(x-1)(x+1)$

$$g(x)=\lambda\left(\frac{x^3}{3}-x\right)+c$$

$$g(-1)=\frac{2\lambda}{3}+c=3\ldots(3)$$

$$g(1)=-\tfrac{2\lambda}{3}+c=-1$$

$$c=1 \quad \text{and} \quad \lambda=3$$

$\therefore \quad g(x)=x^3-3x+1$

$\therefore \quad g(2)=3$

68. **(1)**

$\therefore (p_2-p_1)^2=(q_2-q_1)^2$

$\Rightarrow (p_2+p_1)^2-4p_1p_2=(q_2+q_1)^2-4q_1q_2$

$$\Rightarrow \left(\frac{-b}{a}\right)^2-4\left(\frac{c}{a}\right)=\left(\frac{-b}{c}\right)^2-4\left(\frac{a}{c}\right)$$

$$\Rightarrow \frac{b^2-4ac}{a^2}=\frac{b^2-4ac}{c^2}$$

Since b^2-4ac is the discriminant of both the equations and roots are different

$\therefore \quad b^2\neq 4ac$

$\therefore \quad a^2=c^2$

$\Rightarrow$ a $=$ c (Not possible because two quadratic equations become identical)

or a $=-$c

$\Rightarrow \frac{a}{c}=-1$

69. **(3)** Case (1): $d=7$

${}^6C_3\times{}^3C_3=20$

Case (2): $d=8$

${}^7C_3\times{}^4C_3=140$

Case (3): $d=9$

${}^8C_3\times{}^5C_3=560$

Total $=20+140+560=720$

70. **(2)**

$f(x)=x^4-4x^3-8x^2+a$

$f'(x)=4\left(x^3-3x^2-4x\right)$

$=4x\left(x^2-3x-4\right)$

$=4x(x-4)(x+1)=0$ at $x=-1,0,4$

$f(-1)=a-3\leq 0, a\leq 3$

$f(0)\geq 0\Rightarrow a\geq 0$

$a\in[0,3]$

Sum $=0+1+2+3=6$

71. **(53)** We have $AB=\begin{bmatrix}1 & \frac{3}{2}\\ 1 & 2\end{bmatrix}\begin{bmatrix}4 & -3\\ -2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\mathrm{I}$

$(AB)^1C_1=C_1,(AB)^2C_2=C_2$ and so on.

$\operatorname{tr}(C_r)=r\cdot 3^r+(1-r)\cdot 3^r=3^r$

$\sum_{\mathrm{r}=1}^{50}\operatorname{tr}\left((\mathrm{AB})^{\mathrm{r}}\mathrm{C}_{\mathrm{r}}\right)=\operatorname{tr}\left((\mathrm{AB})^1\mathrm{C}_1\right)+\operatorname{tr}\left((\mathrm{AB})^2\mathrm{C}_2\right)+\ldots\ldots..+\operatorname{tr}\left((\mathrm{AB})^{50}\mathrm{C}_{50}\right)=\mathrm{S}$ (Let)

$S=\operatorname{tr}(C_1)+\operatorname{tr}(C_2)+\ldots.+\operatorname{tr}(C_{50})$

$S=3^1+3^2+\cdots+3^{50}$

$=\frac{3\left(3^{50}-1\right)}{2}$

$\Rightarrow a=3, b=50$

72. **(81)** Slope of AC $=1$

$\Rightarrow$

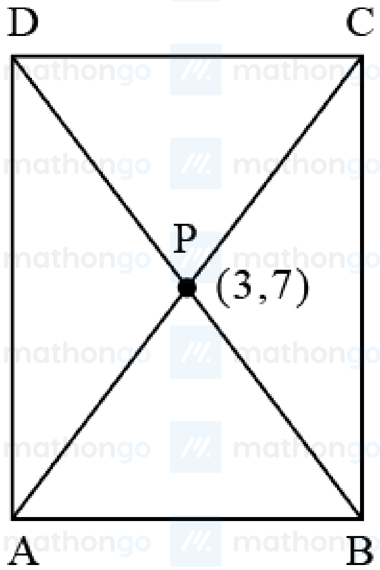
Slope of BD $=-1$

$\because$ Side length $=4$

$\Rightarrow$ Diagonal length $=4\sqrt{2}$

Now, using parametric form,

(i) Coordinates of C, A :



$=\left(3\pm 2\sqrt{2}\cos\frac{\pi}{4}, 7\pm 2\sqrt{2}\sin\frac{\pi}{4}\right)=(3\pm 2, 7\pm 2)$

$=(5,9)$ and $(1,5)$

(ii) Coordinates of D, B:

$\left(3 \pm 2\sqrt{2}\cos\frac{3\pi}{4}, 7 \pm 2\sqrt{2}\sin\frac{3\pi}{4}\right)$

$= (3 \pm 2, 7 \pm 2)$

$= (1, 9)$ and $(5, 5)$

So, we have:

$\mathrm{A} \equiv (x_1, y_1) = (1, 5)$ and $\mathrm{B} \equiv (x_2, y_2) = (5, 5)$

$\mathrm{C} \equiv (x_3, y_3) = (5, 9)$ and $\mathrm{D} \equiv (x_4, y_4) = (1, 9)$

Now, $\frac{y_1 y_2 y_3 y_4}{x_1 x_2 x_3 x_4} = \frac{5\times5\times9\times9}{1\times5\times5\times1} = 81$

73. **(9)** $27^{40} = \left(3^3\right)^{40} = 3^{120}$

$3^{119} = (4-1)^{119}$

$= C_0 4^{119} - C_1 4^{118} + C_2 4^{117} - \ldots\ldots\ldots + C_{118} 4 - C$

$= 4\mathrm{k} - 1$

$\Rightarrow 3^{120} = 3(4\mathrm{k}-1) = 12k - 3$

$= 12(k-1) + 9$

So, the required remainder is 9.

74. **(3)** Unit vector $(\hat{n})$ perpendicular to $\overrightarrow{OA}$ and $\overrightarrow{CB}$

$\hat{n} = \frac{\overrightarrow{\mathrm{CB}} \times \overrightarrow{\mathrm{OA}}}{|\overrightarrow{\mathrm{CB}} \times \overrightarrow{\mathrm{OA}}|} = \frac{(3\alpha-7)\hat{\mathrm{i}} - 4\hat{\mathrm{j}} + (5-\alpha)\hat{\mathrm{k}}}{\sqrt{(3\alpha-7)^2 + 16 + (5-\alpha)^2}}$

As $\overrightarrow{\mathrm{CB}} \times \overrightarrow{\mathrm{OA}} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 1 & \alpha-3 & -1 \\ 1 & 2 & 3 \end{vmatrix}$

Shortest distance $= |\overrightarrow{\mathrm{BA}} \cdot \hat{\mathrm{n}}| = \left|\frac{(7-3\alpha) - 4(2-\alpha) + 2(5-\alpha)}{\sqrt{(3\alpha-7)^2 + 16 + (5+\alpha)^2}}\right|$

$= \sqrt{\frac{3}{2}} \Rightarrow \alpha = 3, \alpha = \frac{9}{7}$

75. **(2)** $2 + \int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

Differentiating w.r.t. x , we get

$f(x)\left(1+x^2\right) = x \Rightarrow f(x) = \frac{x}{1+x^2}$, which is an odd function.

Now $\int_{-x/4}^{\pi/4} \frac{f(x) + x^9 - x^3 + x + 1}{\cos^2 x} dx$

$= \int_{-\pi/4}^{\pi/4} \frac{\frac{x}{1+x^2} + x^9 - x^3 + x}{\cos^2 x} dx + \int_{-\pi/4}^{\pi/4} \sec^2 x dx$

$= 0 + 2$